Period 1, May 7, 2025

Given that $3x - \tan y = 4$, what is $\frac{dy}{dx}$ in terms of y ?

$3 - \sec^2 y \frac{dy}{dx} = 0$

$+ \sec^2 y \frac{dy}{dx}$ $\quad + \sec^2 y \frac{dy}{dx}$

$\Rightarrow \frac{3}{\sec^2 y} = \frac{\cancel{\sec^2} y \frac{dy}{dx}}{\cancel{\sec^2 y}}$

$\frac{3}{\sec^2 y} = \frac{dy}{dx}$

$\frac{\frac{3}{1}}{\frac{1}{\cos^2 y}} = \frac{3}{1} \cdot \frac{\cos^2 y}{1} = 3\cos^2 y = \frac{dy}{dx}$

$$\int \frac{1}{x^2+4x+5}\,dx =$$

A $arctan\,(x+2) + C$

B $arcsin\,(x+2) + C$

C $ln\ |x^2+4x+5| + C$

D $\frac{1}{\frac{1}{3}x^3+2x^2+5x} + C$

$\int \frac{1}{x^2+4x+5}\,dx =$

$x^2+4x+4-4+5 = \underbrace{x^2+4x+4}+1$

$(x+2)^2+1$

$a=1$

$b=4$

$\frac{b}{a} = \frac{4}{2} = 2$

$\left(\frac{b}{2}\right)^2 = 2^2 = 4$

$\int \frac{1}{(x+2)^2+1^2}\,dx$

Let f be the function defined by $f(x) = \sqrt[3]{x}$. What is the approximation for $f\ (10)$ found by using the line tangent to the graph of f at the point $(8,\ 2)$?

Point (8, 2)

$m = ?\ \frac{1}{12}$

$y = x^{\frac{1}{3}}$

$\frac{dy}{dx} = \frac{1}{3} \cdot x^{\frac{1}{3}-1} = \frac{1}{3} x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}} \quad \frac{1}{3\sqrt[3]{8^2}} = \frac{1}{3\sqrt{64}} = \frac{1}{3 \cdot 4}$

$\frac{1}{12}$

Ⓐ $11/6$

Ⓑ $25/12$

Ⓒ $13/6$

Ⓓ $7/3$

$y - 2 = \frac{1}{12}(x - 8)$

$y - 2 = \frac{1}{12}(10 - 8)$

$y - 2 = \frac{2}{12} = \frac{1}{6}$

$y = 2 + \frac{1}{6} = \frac{12}{6} + \frac{1}{6} = \frac{13}{6}$

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critical   critical pt

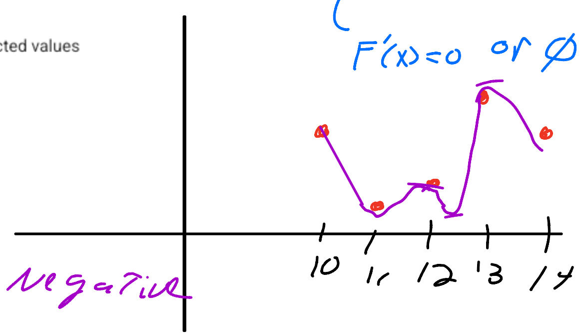
x	10	11	12	13	14
$f(x)$	5	2	3	6	5

dec   in   in   de

The table above gives values of the continuous function f at selected values of x.

If f has exactly two critical points on the open interval $(10,\ 14)$, which of the following must be true?

$f'(x) = 0$ or $\emptyset$



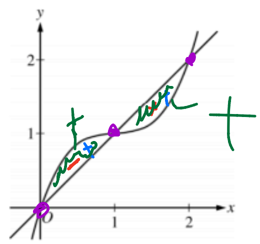
Ⓐ ~~$f(x) > 0$ for all x in the open interval $(10,\ 14)$~~

Ⓑ ~~$f'(x)$ exists for all x in the open interval $(10,\ 14)$~~

Slope is NOT always Negative

Ⓒ $f'(x) < 0$ for all x in the open interval $(10,\ 11)$

Ⓓ $f'(12) \neq 0$



The graphs of the function g and the line $y = x$ are shown in the figure above. The graphs intersect at the points $(0,\ 0)$, $(1,\ 1)$, and $(2,\ 2)$.

Which of the following expressions give the area enclosed by the graphs?

I. $\left|\int_0^2 (x - g(x))dx\right|$ absolute value after they cancel eachother out

II. $\int_0^2 |x - g(x)|dx$ works

III. $\int_0^1 (g(x) - x)dx + \int_1^2 (x - g(x))dx$

(A) II only

(B) III only

(C) I and II only

(D) II and III only

For time $t \geq 1$, the position of a particle moving along the x-axis is given by $p(t) = \sqrt{t} - 2$. At what time t in the interval $1 \leq t \leq 16$ is the instantaneous velocity of the particle equal to the average velocity of the particle over the interval $1 \leq t \leq 16$

Time 15 sec $\left\{ \begin{array}{l} P(1) = \sqrt{1} - 2 = 1 - 2 = -1 \\ P(16) = \sqrt{16} - 2 = 4 - 2 = 2 \end{array} \right\}$ 3 places

Average velocity $= \frac{3}{15} = \frac{1}{5}$

(A) 1

(B) $\frac{121}{25}$

(C) $\frac{25}{4}$

(D) 25

$P(T) = T^{\frac{1}{2}} - 2$

$P'(T) = \frac{1}{2}T^{\frac{1}{2}-1} + 0 = \frac{1}{2\sqrt{T}}$

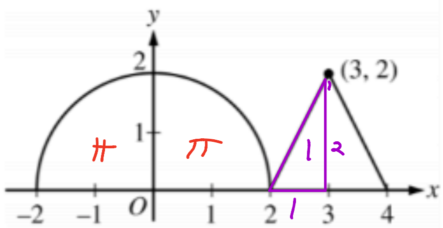
So $\sqrt{T}$ $\quad \frac{1}{5} = \frac{1}{2\sqrt{T}} \cdot 2\sqrt{T} \cdot 5$

$(\sqrt{T})^2 = \left(\frac{5}{2}\right)^2$

$T = \frac{25}{4} = 6\frac{1}{4}$

If $g(0) = 1$, what is $g(3)$?

↳ Start



Graph of g'

Circle $r=2$

Area $\pi r^2 = \pi(2)^2 = 4\pi$

Semicircle $= 2\pi$

$\int_0^3 g'(x)\,dx = \pi + 1$

Change in $g(x)$ from 0 to 3

The graph of g', the first derivative of the function g, consists of a semicircle of radius 2 and two line segments, as shown in the figure above.

$g(3) = g(0) + \int_0^3 g'(x)\,dx = 1 + \pi + 1$

Ⓐ $\pi + 1$

B $\pi + 2$

Ⓒ $2\pi + 1$

Ⓓ $2\pi + 2$

Let f be the function given by $f(x) = x^3 - 6x^2 - 15x$. What is the $\boxed{\text{maximum value}}$ of f on the interval $[0, 6]$?

$=0$

$F'(x) = 3x^2 - 12x - 15$

$= 3(x^2 - 4x - 5)$

$3(x-5)(x+1) = 0$

$x = 5, \cancel{-1}$

x	
0	0
6	-90
5	-100

$F(0) = 0^3 - 6(0)^2 - 15(0)$

$F(6) = \cancel{6^3 - 6 \cdot 6^2} - 15(6) = -90$

$F(5) = 5^3 - 6(5)^2 - 15(5)$

$= 125 - 6 \cdot 25 - 75$

$125 - 150 - 75 = -100$

$\lim\limits_{x \to 0} \frac{4x^2}{e^{4x} - 4x - 1}$ is

$\frac{0}{1-0-1} = \frac{0}{0}$

$\lim\limits_{x \to 0} \frac{4x^2}{e^{4x}-4x-1} = \lim\limits_{x \to 0} \frac{8x}{4e^{4x}-4} = \lim\limits_{x \to 0} \frac{8}{16e^{4x}}$

Plug in 0 $\frac{0}{0} = \phi$ $\quad \frac{8(0)}{4 \cdot e^0 - 4} = \frac{0}{0}$ $\quad \frac{8}{16 \cdot e^0} = \frac{8}{16} = \frac{1}{2}$

(A) 0

(B) 1/2

(C) 8

(D) non existent

Which of the following is a solution to the differential equation $y'' - 4y = 0$?

$4e^{2x} - 4(e^{2x}) = 0$

(A) $y = e^{2x}$ $\quad \frac{dy}{dx} = e^{2x} \cdot 2 = 2e^{2x}$ $\quad \frac{d^2y}{dx^2} = 2e^{2x} \cdot 2 = 4e^{2x}$

(B) ~~$y = 2e^x$~~ $\quad \frac{dy}{dx} = 2e^x$ $\quad \frac{d^2y}{dx^2} = 2e^x \Rightarrow \cancel{2e^x - 4 \cdot 2e^x} = 0$

(C) ~~$y = \sin(2x)$~~ $\quad \frac{dy}{dx} = 2\cos 2x$ $\quad \frac{d^2y}{dx^2} = -4\sin 2x \Rightarrow \cancel{-4\sin 2x - 4\sin^3 x} = 0$

(D) ~~$y = \cos(2x)$~~ $\quad \frac{dy}{dx} = -2\sin 2x$ $\quad \frac{d^2y}{dx^2} = -4\cos 2x \Rightarrow \cancel{-4\cos^2 x - 4\cos^3 x} = 0$

The positive variables p and c change with respect to time t. The relationship between p and c is given by the equation $p^2 = (20-c)^3$.

At the instant when $\frac{dp}{dt} = 41$ and $c = 15$ what is the value of $\frac{dc}{dt}$

$c \le 20$

A. $-\frac{82}{75}$

B. $-\frac{2\sqrt{5}}{3}$

C. $-\frac{3\sqrt{5}}{2}$

D. $-\frac{82\sqrt{5}}{15}$

$p^2 = (20-15)^3$

$p^2 = 5^3$

$p^2 = 125$

$p = \sqrt{125} = 5\sqrt{5}$

$p^2 = (20-c)^3$

$2p\frac{dp}{dT} = 3(20-c)^2 \cdot -\frac{dc}{dT}$

$2 \cdot 5\sqrt{5} \cdot 41 = \underbrace{3(20-15)^2}_{3 \cdot 25} \cdot -\frac{dc}{dT}$

$\frac{410\sqrt{5}}{-75} = \frac{-75\frac{dc}{dT}}{-75}$

$-\frac{5 \cdot 82\sqrt{5}}{5 \cdot 15}$

$p^2 = (20-c)^3$

$p = \sqrt{(20-c)^3} = (20-c)^{3/2}$

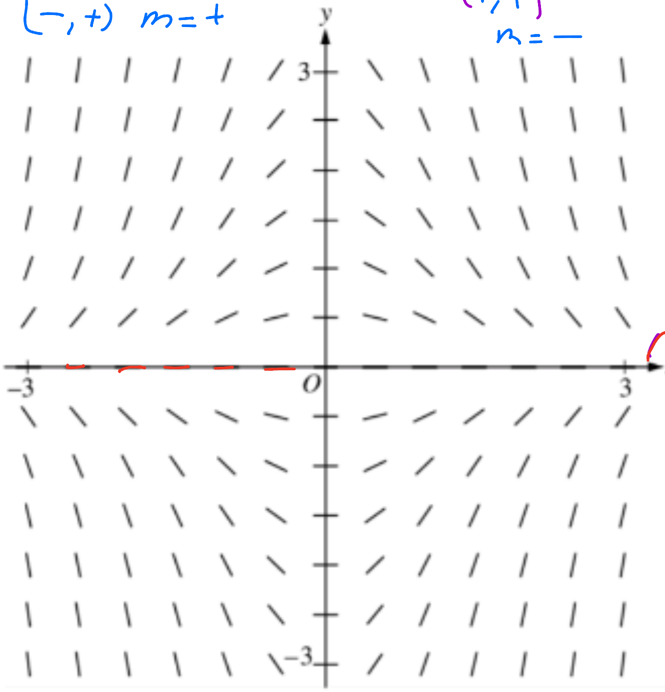
$\frac{dp}{dT} = \frac{3}{2}(20-c)^{\frac{3}{2}-1=\frac{1}{2}} \cdot -\frac{dc}{dT}$

$\frac{3 \cdot 41}{3\sqrt{5}} = \frac{3\sqrt{5}}{\cancel{2}} \cdot - \frac{dc}{dT} \cdot \frac{\cancel{2}}{3\sqrt{5}}$

$-\frac{82\sqrt{5}}{3\sqrt{5} \cdot \sqrt{5}} = \frac{dc}{dT}$

$\frac{-82\sqrt{5}}{3 \cdot 5} = \frac{-82\sqrt{5}}{15} = \frac{dc}{dT}$

(−,+) m=+ (+,+) m=−



y=0 m=0

(+,−) m=+

(−,−) m=−

Ⓐ $\cancel{\frac{dy}{dx}} = x/y$ $\frac{+}{+} = +$

Ⓑ $\frac{dy}{dx} = -\cancel{x/y}$ $-\frac{x}{0} = \phi$

Ⓒ $\cancel{\frac{dy}{dx} = xy}$ $+\cdot+=+$

Ⓓ $\frac{dy}{dx} = -xy$ $-x\cdot 0 = 0$

Using the substitution $u = x + 1$ $\int \frac{x}{\sqrt{x+1}} dx$ is equivalent to

$U = x+1$

$dU = dx$

$x = U - 1$

$$\int \frac{U-1}{\sqrt{U}} dU$$

$$\int \left[\frac{U}{\sqrt{U}} - \frac{1}{\sqrt{U}}\right) dU$$

$$\int \left[\sqrt{U} - \frac{1}{\sqrt{U}}\right) dU = \int \left[U^{\frac{1}{2}} - U^{-\frac{1}{2}}\right] dU$$

(A) $\int \frac{1}{u+1} du$

(B) $\int u^{-1/2} du$

(C) $\int \left(u^{1/2} - u^{-1/2}\right) du$

(D) $(u-1) \int u^{-1/2} du$

When $x = 2e$, $\lim_{h \to 0} \frac{\ln(x+h) - \ln(x)}{h}$ is

$$\lim_{h \to 0} \frac{\ln(x+h) - \ln(x)}{h} = \frac{d}{dx}\left[\ln x\right]$$

$$\frac{1}{x} = \frac{1}{2e}$$

(A) $\frac{1}{2e}$

(B) 1

(C) $\ln(2e)$

(D) nonexistent

A student attempts to solve the differential equation $\frac{dy}{dx} = xy^3$ with the initial condition that $y = 2$ when $x = 0$. The steps of the student's solution are shown below.

Step 1 : $\int \frac{1}{y^3} dy = \int x\, dx$

Step 2 : $\ln |y^3| = \frac{x^2}{2} + C$

Step 3 : $|y^3| = Ke^{x^2/2}$

Step 4 : $|y^3| = 4e^{x^2/2}$

Step 5 : $y = \sqrt[3]{4e^{x^2/2}}$

In which of the following steps does an error first appear?

$\frac{dx}{y^3} \cdot \frac{dy}{dx} = \frac{x\cancel{y^3}}{\cancel{y^3}} \cdot dx$

$\int y^{-3} dy = \int x\, dx$

$-\frac{1}{2} y^{-3+1=-2} = \frac{1}{2}x^2 + C$

$\int \frac{1}{x} dx = \ln|x| + C$

$\int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{1}{-1} x^{-2+1=-1} + C$

$\frac{-1}{x} + C$

A particle moves along the x-axis so that at any time t, $t \geq 0$, its acceleration is $a(t) = -4\sin(2t)$. If the velocity of the particle at $t = 0$ is $v(0) = 7$ and its position at $t = 0$ is $x(0) = 0$, then its position at time t is $x(t) =$

A) $\sin(2t) + 5t$

B) $\sin(2t) + 7t$

C) $\sin(2t) + 9t$

D) $16\sin(2t) + 7t$

$V(T) = 2\cos 2T + 5$

$X(T) = \int V(T)\, dT$

$\int (2\cos 2T + 5)\, dT$

$\int a(T) = V(T)$

$\int -4\sin 2T\, dT = \int -4\sin u \cdot \frac{du}{2} = \int -2\sin u\, du$

$2T = u$

$2\, dT = du$

$dT = \frac{du}{2}$

$-2(-\cos u) + C$

$V(T) = 2\cos 2T + C$

$V(0) = 2\cos 2\cdot 0 + C = 2\cdot 1 + C = 7$

$C = 5$

$X(T) = \sin 2T + 5T + C$

$X(0) = 0 = \sin 2\cdot 0 + 5(0) + C$

$0 = 0 + 0 + C$

$0 = C$

$X(T) = \sin 2T + 5T$

Q28 Limits at infinity

$\lim_{x \to -\infty} \frac{3+2^x}{4-5^x}$ is

$$\frac{3+2^{-1,000,000}}{4-5^{-1,000,000}} = \frac{3+\frac{1}{2^{1000000}}}{4-\frac{1}{5^{1000000}}} = \frac{3+0}{4-0} = \frac{3}{4}$$

(A) $-2/5$

(B) 0

(C) $3/4$

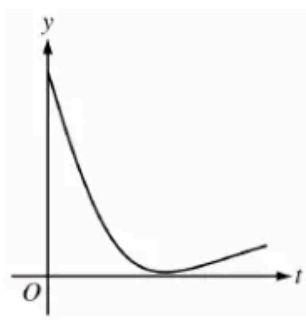
(D) non existent

is on notes From 5/3

5. During a chemical reaction, the function $y = f(t)$ models the amount of a substance present, in grams, at time t seconds. At the start of the reaction $(t = 0)$, there are 10 grams of the substance present. The function $y = f(t)$ satisfies the differential equation $\frac{dy}{dt} = -0.02y^2$.

(a) Use the line tangent to the graph of $y = f(t)$ at $t = 0$ to approximate the amount of the substance remaining at time $t = 2$ seconds.

(b) Using the given differential equation, determine whether the graph of f could resemble the following graph. Give a reason for your answer.



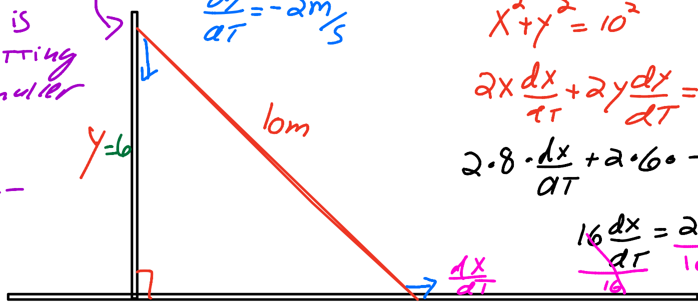
(c) Find an expression for $y = f(t)$ by solving the differential equation $\frac{dy}{dt} = -0.02y^2$ with the initial condition $f(0) = 10$.

(d) Determine whether the amount of the substance is changing at an increasing or a decreasing rate. Explain your reasoning.

11. A ladder 10 meters long is leaning against a vertical wall with its other end on the ground. The top end of the ladder is sliding down the wall. When the top end is 6 meters from the ground it is sliding down at 2 m/sec. How fast is the bottom moving away from the wall at this instant?

y is getting smaller so $\frac{dy}{dT} = -$

$\frac{dy}{dT} = -2\,m/s$



$\frac{dx}{dT}$

$x^2+y^2=10^2$

$2x\frac{dx}{dT}+2y\frac{dy}{dT}=0$

$2\cdot 8\cdot\frac{dx}{dT}+2\cdot 6\cdot -2=0$

$16\frac{dx}{dT}=24$; $\frac{16}{16}$, $\frac{24}{16}$

$\frac{dx}{dT}=\frac{24}{16}\,m/s=\frac{3}{2}\,m/s$

$x^2+6^2=10^2$

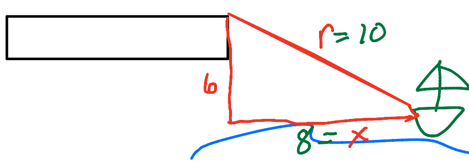
$x^2+36=100$

$x^2=64$

$x=8$

8. A boat is being pulled into a dock by attached to it and passing through a pulley on the dock, positioned 6 meters higher than the boat. If the rope is being pulled in at a rate of 3 meters/sec, how fast is the boat approaching the dock when it is 8 meters from the dock?

$\frac{dx}{dT}=?$



$6^2+x^2=r^2$

$\frac{dr}{dT}=-3\,m/s$

$0+2x\frac{dx}{dT}=2r\frac{dr}{dT}$

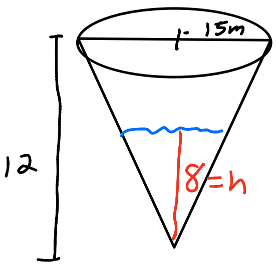
$2\cdot 8\cdot\frac{dx}{dT}=2\cdot 10\cdot -3 \Rightarrow 16\frac{dx}{dT}=-\frac{60}{16}$; $\frac{16}{16}$

$6^2+8^2=r^2 \Rightarrow 36+64=r^2 \Rightarrow 100=r^2 \Rightarrow r=10$

$\frac{dx}{dT}=-\frac{15}{4}\,m/s$

10. A water tank has the shape of an inverted right-circular cone, with radius at the top 15 meters and depth 12 meters. Water is flowing into the tank at the rate of 2 cubic meters per minute. How fast is the depth of water in the tank increasing at the instant when the depth is 8 meters?

$\frac{dh}{dT}$ $\qquad \frac{dV}{dT} = 2\ m^3/min$



$\frac{r}{h} = \frac{15}{12}$

$12r = 15h$

$r = \frac{15h}{12} = \frac{5h}{4}$

$V = \frac{1}{3}\pi r^2 \cdot h$

$V = \frac{1}{3}\pi \left(\frac{5h}{4}\right)^2 \cdot h$

$V = \frac{1}{3}\pi \cdot \frac{25h^2}{16} \cdot h$

$V = \frac{25\pi}{48} h^3$

$\frac{dV}{dT} = \frac{25\pi}{48} \cdot 3h^2 \frac{dh}{dT}$

$2\ m^3/min = \frac{25\pi}{48} \cdot 3(8m)^2 \frac{dh}{dT}$

$\frac{2m^3/min}{100\pi m^2} = \frac{\cancel{100\pi m^2}\frac{dh}{dT}}{\cancel{100\pi m^2}}$

$\frac{\frac{2m^3}{min}}{\frac{100\pi m^2}{1}} = \frac{2m^3}{min} \cdot \frac{1}{100\pi m^2}$

$2\ m^3/min = \frac{25\pi}{48} \cdot 3(8)^2 \cdot \frac{dh}{dT}$

$2 = \frac{25\pi \cdot 3 \cdot 64}{48} \frac{dh}{dT}$

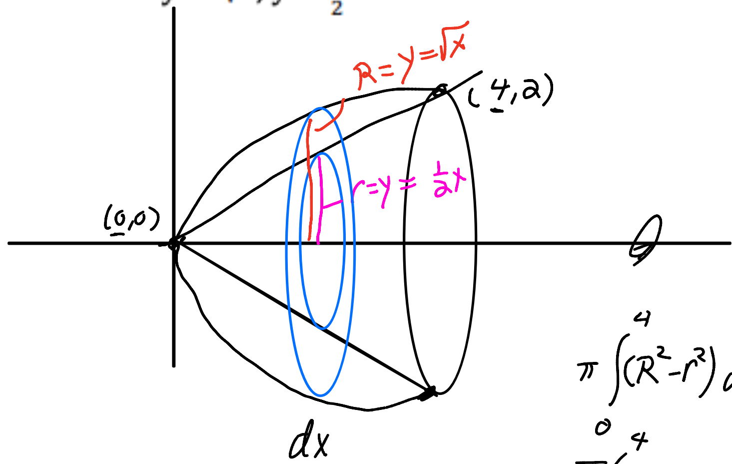
$2 = \frac{25\pi \cdot \cancel{3} \cdot \cancel{16} \cdot 4}{\cancel{3} \cdot \cancel{16}} \frac{dh}{dT}$

$2\ m^3/min = 100\pi\ m^2 \cdot \frac{dh}{dT}$

$\frac{2}{100\pi}\ m/min = \frac{dh}{dT}$

1. Find the volume of the solid generated by revolving the area bounded by the following about the x-axis using the disk/washer and shell methods.

$y = \sqrt{x}, y = \frac{1}{2}x$



$(\sqrt{x})^2 = \left(\frac{1}{2}x\right)^2$

$x = \frac{x^2}{4}$ $-x$ $-x$

$0 = \frac{x^2}{4} - x = x\left(\frac{x}{4} - 1\right)$

$x = 0$ or $x = 4$

$\pi \int_0^4 (R^2 - r^2)\,dx$

$\pi \int_0^4 \left[(\sqrt{x})^2 - \left(\frac{1}{2}x\right)^2\right] dx$

$\pi \int_0^4 \left(x - \frac{x^2}{4}\right) dx = \pi \left[\frac{1}{2}x^2 - \frac{x^3}{12}\right]_0^4$

$\pi \left[\left(\frac{1}{2}(4)^2 - \frac{4^3}{12}\right) - \cancel{\left(\frac{1}{2}(0)^2 - \frac{0^3}{12}\right)}\right]$

$\pi \left[\frac{16 \cdot 6}{2 \cdot 6} - \frac{64}{12}\right]$

$\pi \left[\frac{96}{12} - \frac{64}{12}\right] = \frac{32}{12}\pi = \frac{8}{3}\pi$

3. Find the volume of the solid generated by revolving the area bounded by the following about the line $y = -1$ using the disk/washer and shell methods.

$y = x, x = 4, y = 0$